Exercise 1

There is a small island somewhere in the Caribbean. Passenger ships (200 per year) travel from the continent to the island and continue their route to other islands afterwards. Only 2 routes are known through the sea. The danger of sinking is 10% with the short route, and 5% with the long route. About 20% of the ships decided to take the shorter route.

Q1: how many sinking ships do we expect in a year?

Q2: a sunken ship is reported but we don’t know where it is. Which route should we check first?

Exercise 2

After your yearly check-up, the doctor comes up with your test results. Unfortunately you tested positive for a serious disease and the test is 99% accurate (i.e. the probability of testing positive given that you have the disease is 0.99, as is the probability of testing negative given that you don’t have the disease). This disease usually strikes only 1 in 10000 people.

Q1: Should you be worried?

Q2: Can you consider it good news that the test is 99% accurate?
Exercise 3

The following questions ask you to prove more general versions of probability rules with respect to some background evidence E.

Q1: Prove the conditional version of the product rule:
\[ p(A, B \mid E) = p(A \mid B, E)p(B \mid E) \]

Q2: Prove the conditional version of Bayes rule:
\[ p(A \mid B, E) = \frac{p(B \mid A, E)p(A \mid E)}{p(B \mid E)} \]

Exercise 4

This problem investigates the way in which conditional independence relationships affect the amount of information needed for probabilistic calculations.

Q1: We wish to calculate \( p(H \mid E_1, E_2) \), and we have no conditional independence information. Which of the following sets of quantities is/are sufficient for the calculations?

1. \( p(E_1, E_2), p(H), p(E_1 \mid H), p(E_2 \mid H) \)
2. \( p(E_1, E_2), p(H), p(E_1, E_2 \mid H) \)
3. \( p(E_1 \mid H), p(E_2 \mid H), p(H) \)

Q2: We now know that \( E_1 \) and \( E_2 \) are conditionally independent given \( H \). Which of the 3 sets is/are sufficient?
Bayesian networks

Exercise 1

Q1: what is the joint distribution of all these variables?

Q2: what can we say about R, T and E? About W and P?

Q3: assuming that the appropriate conditional probabilities are known:
   - find the probability to get bored when the temperature outside is cold
   - find the probability that the temperature is cold when you are bored
   - find the probability to have to work when you are sick
   - what can you deduce from it?

Q4: using variable elimination:
   - find the probability to be stuck at home
   - find the probability to be sick when you had to work

R: rain ∈ {0,1}
T: temperature ∈ {cold,warm}
S: sick ∈ {0,1}
E: exam ∈ {0,1}
P: party ∈ {0,1}
W: must work ∈ {0,1}
H: must stay home ∈ {0,1}
B: bored ∈ {0,1}
Exercise 2

Q1: Compute a minimal I-map for P using the ordering \{D,I,S,G,L\}.

Q2: Compute a minimal I-map for P using the ordering \{L,D,S,I,G\}.

Exercise 3

Q1: Compute a minimal I-map for P using the ordering \{X,Y,Z,W,I\}.

Q2: Using the PC algorithm, compute the perfect map for P.